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TO'PLAMNING XARAKTERISTIK FUNKSIYASINING ALGEBRAIK XOSSALARI

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Annotatsiya: *Haqiqiy o'zgaruvchili funksiyalar nazariyasida xarakteristik funksiyalar muhim rol o'ynaydi. Ushbu maqolada, yuqoridagidan farqli ravishda ushbu funksiyalar sinfini algebraik sistema elementlari sifatida muhim xossalarini o'rganilgan, ya'ni halqalarning nolning bo'luvchilari va idempotent elementlar to'plami halqadagi qo'shish amaliga nisbatan yopiq sistema hosil qilmasligi ko'rsatilgan.*

Kalit so'zlar: *Xarakteristik funksiya, to'plamlar ustida amallar, to'plamlar sistemasi, halqa, halqaning idempotent elementi, nolning bo'luvchilari.*

Kirish

Aytaylik bizga X to'plam berilgan bo'lsin. Ushbu to'plamning barcha qism to'plamlari sistemasini 2^X bilan belgilaylik . X ning ixtiyoriy $A\subset X, B\subset X$ qism to'plamlarining xarakteristik funksiyasi deb quyidagi funksiyaga aytamiz:

$$\chi_A(x): X \to \square,\ \chi_A(x)=\begin{cases}1, & x\in A\\ 0, & x\notin A\end{cases} \text{ va } \chi_B(x): X \to \square\ \ \chi_B(x)=\begin{cases}1, & x\in B\\ 0, & x\notin B\end{cases}. \text{ [2]}$$

Xarakteristik funksiyalarni quyidagi muhim bir xossalariga e'tibor bersak:

(i) $A=B \Leftrightarrow \chi_A=\chi_B$.

$\chi_A=\chi_B \Leftrightarrow \forall x\in X: \chi_A(x)=\chi_B(x) \Leftrightarrow (x\in A)\Leftrightarrow(x\in B),(x\notin A)\Leftrightarrow(x\notin B)$, yoki qisqacha qilib aytsak ixtiyori ikkitra to'plam teng bo'lishi uchun ularda aniqlangan xarakteristik funksiyalari aynan teng bo'lishiu zarur va yetarlidir .□

(ii) $\chi_X\equiv 1,\ \chi_\varnothing\equiv 0,\ \chi_A(x)\cdot\chi_A(x)\equiv\chi_A(x).$

(iii) $\chi_{A\cap B}(x)\equiv\chi_A\cdot\chi_B$

Bu xossani isbotlashda umiy holda quyidagi ikki xil mulohazalardan biri har doim o'rinli:

- $\chi_{A\cap B}(x)=1 \Leftrightarrow x\in A\bigcap B \Leftrightarrow (x\in A)\wedge(x\in B) \Leftrightarrow (\chi_A=1)\wedge(\chi_B=1) \Leftrightarrow \chi_A\cdot\chi_B=1;$
- $\chi_{A\cap B}(x)=0 \Leftrightarrow x\notin A\bigcap B \Rightarrow \neg((x\in A)\wedge(x\in B)) \Rightarrow (\chi_A=0)\vee(\chi_B=0) \Leftrightarrow \chi_A\cdot\chi_B=0.$

Ikkala xulosani jamlab $\chi_{A\cap B}(x)\equiv\chi_A\cdot\chi_B$ o'rinli ekanini olamiz □

(iv) $\chi_{A\cup B}(x)\equiv\chi_A(x)+\chi_B(x)-\chi_A(x)\cdot\chi_B(x)$

Xuddi (iii) xossani isbotida bo'lgani kabi quyidagi ikki mulohazadan biri har doim o'rinli ekanidan foydalanamiz:

- $\chi_{A\cup B}(x)=1 \Leftrightarrow x\in A\bigcup B \Leftrightarrow (x\in A)\vee(x\in B) \Leftrightarrow (\chi_A=1)\vee(\chi_B=1) \Leftrightarrow \left[\begin{array}{l}(\chi_A=1),(\chi_B=0)\\(\chi_A=0)\vee(\chi_B=1)\\(\chi_A=1)\vee(\chi_B=1)\end{array}\right. \Leftrightarrow$

$$\Leftrightarrow \left[\begin{array}{l}(\chi_A+\chi_B=1)\\(\chi_A+\chi_B=1)\\(\chi_A+\chi_B=2)\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\chi_A+\chi_B-\chi_A\cdot\chi_B=1\\\chi_A+\chi_B-\chi_A\cdot\chi_B=1;\\\chi_A+\chi_B-\chi_A\cdot\chi_B=1\end{array}\right.$$

- $\chi_{A\cup B}(x)=0 \Leftrightarrow x\notin A\bigcup B \Leftrightarrow \neg((x\in A)\vee(x\in B)) \Leftrightarrow ((x\notin A)\wedge(x\notin B)) \Leftrightarrow$
$\Leftrightarrow (\chi_A=0)\wedge(\chi_B=0) \Leftrightarrow$

$\Leftrightarrow \chi_A+\chi_B-\chi_A\cdot\chi_B=0$

Yuqoridagi ikki mulohazani umulashtirib $\chi_{A\cup B}(x)\equiv\chi_A(x)+\chi_B(x)-\chi_A(x)\cdot\chi_B(x)$ ekanini olamiz.□

(v) $\chi_{X\setminus A}(x)\equiv 1-\chi_A(x)$

- $\chi_{X\setminus A}(x)=1 \Leftrightarrow x\in X\setminus A \Leftrightarrow (x\in X)\wedge(x\notin A) \Leftrightarrow (\chi_X\equiv 1)\wedge(\chi_A\equiv 0) \Leftrightarrow 1-\chi_A=1 \Leftrightarrow 1-\chi_A=1$

- $\chi_{X\setminus A}(x)=0 \Leftrightarrow x\notin X\setminus A \Leftrightarrow \neg((x\in X)\wedge(x\notin A)) \Leftrightarrow ((x\notin X)\vee(x\in A)) \Leftrightarrow$
$\Leftrightarrow (\chi_X\equiv 0)\vee(\chi_A\equiv 1) \Leftrightarrow 0-\chi_A=-1 \Leftrightarrow 1-\chi_A=1-1=0 \Leftrightarrow \chi_{X\setminus A}(x)\equiv 1-\chi_A(x)$

$\chi_{X\setminus A}(x)\equiv 1-\chi_A(x)$□

(vi) $\chi_{A\setminus B}(x)\equiv\chi_A(x)-\chi_A(x)\chi_B(x)$

Ushbu xossani isbotlashda to'plamlar ustidagi ushbu $A\setminus B=A\cap(X\setminus B)$ ayniyat orqali ko'rsatamiz

$A\setminus B=A\cap(X\setminus B)$ ayniyat o'rinli, u holda (i) xossaga ko'ra, $\chi_{A\setminus B}(x)\equiv\chi_{A\cap(X\setminus B)}(x)\equiv$[(iii) xossaga ko'ra] $\equiv\chi_A(x)\cdot\chi_{X\setminus B}(x)\equiv\chi_A(x)\cdot(1-\chi_B(x))\equiv\chi_A(x)-\chi_A(x)\cdot\chi_B(x)$.□

(vii) $\chi_{A\Delta B}(x)\equiv\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\cdot\chi_B(x)$

Ushbu xossani isbotlashda to'plamlar ustidagi ushbu $A\Delta B=(A\setminus B)\bigcup(B\setminus A)$ ayniyat orqali ko'rsatamiz

$A\Delta B=(A\setminus B)\bigcup(B\setminus A)$ ayniyat o'rinli, u holda (i) xossaga ko'ra, $\chi_{A\Delta B}(x)\equiv\chi_{(A\setminus B)\cup(B\setminus A)}(x)\equiv$ [(iv) xossaga ko'ra] $\equiv\chi_{A\setminus B}(x)+\chi_{B\setminus A}(x)-\chi_{A\setminus B}(x)\cdot\chi_{B\setminus A}(x)\equiv$

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$$\left(\chi_A(x)-\chi_A(x)\cdot\chi_B(x)\right)+\left(\chi_B(x)-\chi_B(x)\cdot\chi_A(x)\right)-$$

$$-\left(\chi_A(x)-\chi_A(x)\cdot\chi_B(x)\right)\cdot\left(\chi_B(x)-\chi_B(x)\cdot\chi_A(x)\right)\equiv$$

$$\chi_A(x)-\chi_A(x)\cdot\chi_B(x)+\chi_B(x)-\chi_B(x)\cdot\chi_A(x)-$$

$$-\chi_A(x)\cdot\chi_B(x)-\chi_A(x)\cdot\chi_A(x)\cdot\chi_B(x)-\chi_A(x)\cdot\chi_B(x)\cdot\chi_B(x)+\chi_A(x)\cdot\chi_B(x)\cdot\chi_A(x)\cdot\chi_B(x)\equiv$$

[(ii)ga ko'ra]

$$\chi_A(x)+\chi_B(x)-2\cdot\chi_B(x)\cdot\chi_A(x)-\chi_A(x)\cdot\chi_B(x)-2\cdot\chi_A(x)\cdot\chi_B(x)+\chi_A(x)\cdot\chi_B(x)\equiv$$

$$\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\cdot\chi_B(x).\square$$

Yuqorida keltirilgan 7 ta xossadan foydalanib, ixtiyoriy to'plamlar ustidagi ayniyatlarni o'rinli yoki o'rinli emas ekanligini isbotlashda foydalanishimiz mumkin.

Masalan, quyidagi to'plamlar ustida berilgan ikkita munosabatlarni qaraylik:

a) $(A\setminus B)\setminus C\neq A\setminus(B\setminus C)$; b) $A\bigcup B=(A\Delta B)\Delta(A\bigcap B)$

(a):

- $\chi_{(A\setminus B)\setminus C}(x)=\chi_{(A\setminus B)}(x)-\chi_{(A\setminus B)}(x)\cdot\chi_C(x)\equiv\left(\chi_A(x)-\chi_A(x)\cdot\chi_B(x)\right)-\left(\chi_A(x)-\chi_A(x)\cdot\chi_B(x)\right)\cdot\chi_C($
  $\equiv\chi_A(x)-\chi_A(x)\cdot\chi_B(x)-\chi_A(x)\cdot\chi_C(x)-\chi_A(x)\cdot\chi_B(x)\cdot\chi_C(x)$
- $\chi_{A\setminus(B\setminus C)}(x)=\chi_A(x)-\chi_A(x)\cdot\chi_{B\setminus C}(x)\equiv\chi_A(x)-\chi_A(x)\cdot\left(\chi_B(x)-\chi_B(x)\chi_C(x)\right)\equiv$

$$\equiv\chi_A(x)-\chi_A(x)\cdot\chi_B(x)-\chi_A(x)\cdot\chi_B(x)\chi_C(x)$$

Yuqoridagi 2 ta tengliklardan ko'rinib turibdiki, $A\setminus(B\setminus C)$ va $(A\setminus B)\setminus C$ to'plamlarning xarakteristik funksiyalari aynan teng bo'lmas ekan. Demak , (i) xossaga ko'ra, ushbu to'plamlar o'zaro teng emas ekan.

(b):

- (iv) xossaga ko'ra bevosita ushbu ayniyatni olamiz: $\chi_{A\cup B}(x)\equiv\chi_A(x)+\chi_B(x)-\chi_A(x)\cdot\chi_B(x)$
- Endi $(A\Delta B)\Delta(A\bigcap B)$ ning xarakteristik funksiyasini qaraymiz:

$$\chi_{(A\Delta B)\Delta(A\cap B)}(x)\equiv\chi_{(A\Delta B)}(x)+\chi_{(A\cap B)}(x)-2\cdot\chi_{(A\Delta B)}(x)\cdot\chi_{(A\cap B)}(x)\equiv\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\chi_B(x)+$$

$$+\left(\chi_A(x)\chi_B(x)\right)-2\cdot\left(\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\chi_B(x)\right)\cdot\left(\chi_A(x)\chi_B(x)\right)\equiv$$

$$\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\chi_B(x)+$$

$$\chi_A(x)\chi_B(x)-2\cdot\left(\chi_A(x)\chi_B(x)+\chi_A(x)\chi_B(x)-2\cdot\chi_A(x)\chi_B(x)\right)\equiv$$

$$\equiv\chi_A(x)+\chi_B(x)-2\cdot\chi_A(x)\chi_B(x)+\chi_A(x)\chi_B(x)-2\cdot 0\equiv\chi_A(x)+\chi_B(x)-\chi_A(x)\chi_B(x)$$

$\chi_{(A\Delta B)\Delta(A\cap B)}(x)\equiv\chi_{A\cup B}(x)\Rightarrow$ [(i) ga ko'ra] $\Rightarrow(A\Delta B)\Delta(A\bigcap B)=A\bigcup B.\square$

Asosiy qism.

Endi abstrakt algebradagi halqaga va unga oid bo'lgan tushunchalarni keltirib o'tsak. Aytaylik, R to'plam va ushbu to'plamda aniqlangan "$+$" va "$\cdot$" binar amallar berilgan bo'lsin. Agar ushbu amallar quyidagi:

1. $(R,+)$-kommutativ gruppa;
2. $(R,\cdot)$-yarim gruppa;
3. $\forall a,b,c\in R$ elementlar uchun $(a+b)c=ac+bc$ (chap distributivlik),

$a(b+c)=ab+ac$ (o'ng distributivlik);

shartlarni qanoatlantirsa , u holda, $(R,+,\cdot)$ tartiblangan uchlikka "**halqa"** deyiladi. Bu yerda, $(R,+)$ gruppaning neytral elementi halqaning "0" nol elementi deyiladi, $(R,\cdot)$ yarim gruppaning neytral elementi esa, halqaning "1" birlik elementi deyiladi(agar u mavjud bo'lsa!);

Ta'rif 1: Agar $a\in R$ element uchun $a^2=a$ tenglik o'rinli bo'lsa, u holda a elementga "**idempotent element"** deyiladi.

Ta'rif 2: Agar $a\in R\setminus\{0\}$ element uchun $\exists b\in R\setminus\{0\}$ element topilib, $ab=0$ tenglik o'rinli bo'lsa, u holda a elementga "**nolning bo'luvchisi"** deyiladi.

Halqalar ustida berilgan ayrim muammolarning yechimini xarakteristik funksiyalar orqali hal qilishimiz mumkin. Buning uchun dastlab, haqiqiy sonlarda aniqlangan barcha funksiyalar sinfi $F(\mathbb{R})$ ni, funksiyalar ustidagi odatdgi "$+$" va "$\cdot$" amallar bilan qaraylik:

$$\forall f,g,\in F(\mathbb{R}),\forall x\in\mathbb{R}:(f+g)(x)\overset{def}{=}f(x)+g(x),\ (f\cdot g)(x)\overset{def}{=}f(x)\cdot g(x)$$

$(F(\mathbb{R}),+,\cdot)$ sistema halqa tashkil qiladi [1].

Endi quyidagi masalalarni qaraylik.

a) Halqaning idempotent elementlarininng yig'indisi ham idempotent element bo'ladimi?

b) Halqadagi nolning bo'luvchilarining yig'indisi ham nolning bo'luvchisi bo'ladimi?

Yechim:

a) Aytaylik, haqiqiy soblar to'plamidagi biror to'plam berilgan bo'lsin $A,B\in 2^{\mathbb{R}},\{a\}\subset A\cap B$ $(F(\mathbb{R}),+,\cdot)$ halqadagi quyidagi ikki elementini qaraylik: $\chi_B(x),\chi_A(x)\in F(\mathbb{R})$. (ii) xossaga ko'ra, ixtiyoriy xarakteristik funksiyalar idempotent bo'ladi:

$$\chi^2_A(x)\equiv\chi_A(x),\chi^2_B(x)\equiv\chi_B(x)$$

<u>Quyida ularning yig'indisini idempotentlikka tekshiramiz:</u>

$(\chi_A(x)+\chi_B(x))^2\equiv\chi^2_A(x)+\chi^2_B(x)+2\chi_{A\cap B}(x)\Rightarrow$ [(ii),(iii) xossalarga ko'ra]

$\left(\chi_A(x)+\chi_B(x)\right)^2 \equiv \chi^2{}_A(x)+\chi^2{}_B(x)+2\chi_{A\cap B}(x) \Rightarrow$ ekanini olamiz.

Aytaylik, $x=5,\ A=[3,6]$ bo'lsin, u holda, yuqoridagi ayniyatga ko'ra,

$\left(\chi_{[3,6]}(5)+\chi_{[3,5]}(5)\right)^2=\chi_{[3,6]}(5)+\chi_{[3,5]}(5)+2\chi_{[5,6]}(5)=4$ ekanligini olamiz. Boshqa tarafdan, $\chi_{[3,6]}(5)+\chi_{[3,5]}(5)=2$ tenglik o'rinli, ya'ni

$$\begin{cases}\chi_{[3,6]}(5)+\chi_{[3,5]}(5)=2\\ \left(\chi_{[3,6]}(5)+\chi_{[3,5]}(5)\right)^2=4\end{cases} \Rightarrow \left(\chi_{[3,6]}(5)+\chi_{[3,5]}(5)\right)^2 \neq \chi_{[3,6]}(5)+\chi_{[3,5]}(5).$$

Bundan ma'lum bo'ladiki, $\left(\chi_{[3,6]}(5)+\chi_{[3,5]}(5)\right)^2 \neq \chi_{[3,6]}(5)+\chi_{[3,5]}(5).$ ayniyat o'rinli emas ekan. Demak a) masala barcha halqalarda ham ijobiy hal qilinmaydi ya'ni ixtiyoriy halqada idempotent elementlarning yig'indisi idempotent bo'lmaydi.

b) $\left(F(\mathbb{R}),+,\cdot\right)$ halqadagi quyidagi ikki elementini qaraylik: $\chi_{[2,5]}(x), \chi_{\mathbb{R}\setminus[2,5]}(x) \in F(\mathbb{R})$. Bu elementlar nol emas: $\chi_{[2,5]}(x)\neq 0,\ \chi_{\mathbb{R}\setminus[2,5]}(x)\neq 0$. Lekin, ularning ko'paytmasi nolga teng:

$\chi_{[2,5]}(x)\cdot\chi_{\mathbb{R}\setminus[2,5]}(x) \equiv$ [(iii) xossaga ko'ra] $\equiv \chi_{[2,5]\cap(\mathbb{R}\setminus[2,5])}(x) \equiv \chi_{\varnothing}(x) \equiv 0$.

Demak, $\chi_{[2,5]}(x), \chi_{\mathbb{R}\setminus[2,5]}(x) \in F(\mathbb{R})$ elementlar nolning bo'luvchilari ekan. Endi ularning yig'indisini nolning bo'luvchisi bo'lishini tekshiramiz.

$\chi_{[2,5]}(x)+\chi_{\mathbb{R}\setminus[2,5]}(x) \equiv$ [(iv) xossaga ko'ra] $\equiv \chi_{[2,5]\cup(\mathbb{R}\setminus[2,5])}(x)+\chi_{[2,5]\cap(\mathbb{R}\setminus[2,5])}(x) \equiv \chi_{\mathbb{R}}(x)+\chi_{\varnothing}(x) \equiv 1$

$\chi_{[2,5]}(x)+\chi_{\mathbb{R}\setminus[2,5]}(x) \equiv 1$. Halqalarning birlik elementi hech qachon nolning bo'luvchisi bo'lmaydi[1]. Shu sababli, $\chi_{[2,5]}(x)+\chi_{\mathbb{R}\setminus[2,5]}(x)$ element nolning bo'luvchisi emas. Demak b) masala ham barcha halqalarda ham ijobiy hal qilinmaydi ya'ni ixtiyoriy halqada nolning bo'luvchilarining yig'indisi nolning bo'luvchisi bo'lmaydi.

Xulosa

Xulosa qilib shuni aytamiz mumkinki, halqanining nolni bo'luvchilari va idempotent elementlaridan tuzilgan sistema halqadagi qo'shish amaliga nisbatan yopiq sistema tashkil qilmaydi. Ushbu tasdiqni biz xarakteristik funksiyaning xossalaridan foydalangan holda, misol keltirish orqali, har doim ham o'rinli bo'lmasligini ko'satdik

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